

# Infrared He-Xe laser interferometry for measuring length

Hirokazu Matsumoto

A technique is described for determining absolute lengths up to 3 m from measurement of the interference fringe fractions for multiple wavelengths. The required accuracies of the wavelength ratio and fringe fraction reading are analyzed. To satisfy the accuracies, a low-cost simply stabilized He-Xe laser, operating simultaneously at two wavelengths, 3.51 and 3.37  $\mu\text{m}$ , was developed. Using this laser the preliminary measurement of the wavelength ratio was made interferometrically with the length standard of a gauge block.

## I. Introduction

In precise measurement of length it is convenient to use the method of excess fractions with multiple wavelength interference. At present, cadmium, mercury, or krypton atoms are such a light source, but they cannot be applied to the measurement of lengths greater than several tens of centimeters because of their poor coherency.<sup>1</sup>

A single longitudinal mode laser has very good coherence,<sup>2</sup> but in the visible region when oscillating at multiple wavelengths it is difficult to maintain a single longitudinal mode,<sup>3,4</sup> and several laser sources, each of which is stabilized to a high degree of accuracy, are needed for achieving the method of excess fractions. Therefore, visible multiple wavelength interferometry is difficult for long length measurement. A He-Xe laser oscillates at several wavelengths in the 3- $\mu\text{m}$  region<sup>5</sup> and operates in a single longitudinal mode without a mode selector because of its high gain and long wavelength. In particular, the laser lines at 3.51 and 3.37  $\mu\text{m}$  are, it is hoped, for interferometric length measurement because of their different transitions between atomic energy levels and their good atmospheric transmission. Optical elements and detectors in this wavelength region are of low cost. The 3.51- $\mu\text{m}$  line of the He-Xe laser has been frequency stabilized with an accuracy of better than  $1 \times 10^{-9}$  by using a  $\text{H}_2\text{CO}$  absorption line.<sup>6</sup>

Multiple wavelength laser interferometry for length measurement was studied for reading interference orders.<sup>7,8</sup> In the measurement of long length, the accuracy of the laser wavelength used is also important. Although an overall accuracy of  $1 \times 10^{-7}$  is sufficient for length measurement, much higher accuracy is required in the laser wavelength for achieving the method of excess fractions. However, the required accuracy is not in either of the two wavelengths but in their ratio. A simple method, which uses a two-wavelength simultaneously oscillating He-Xe laser, is proposed for measuring absolute lengths up to 3 m. The stabilization of the wavelength is easily done by a method based on the dependence of output power on cavity tuning.<sup>9-12</sup>

## II. Analysis

### A. Interferometry

Let us consider the problem of determining the length from the interference order fractions of the fringes obtained with two wavelengths  $\lambda_i$ ,  $i = 1, 2$ . The interference order equation for length  $L$  is simply given by

$$L = (M_i + m + \epsilon_i) \cdot \lambda_i / 2, \quad (1)$$

where  $M_i$  and  $m$  are the integral numbers of the interference orders known by other means and determined by the method of excess fractions, respectively, and  $\epsilon_i$  is the measured fractions of the interference orders. From Eq. (1) the following equation is obtained:

$$m = [(M_2 + \epsilon_2) - (M_1 + \epsilon_1) \cdot f] / (f - 1), \quad (2)$$

where  $f$  is the wavelength ratio  $\lambda_1/\lambda_2$ . The uncertainty on  $m$  will be such that

$$\delta m = \frac{2f}{f-1} \delta \epsilon + \frac{2L}{\lambda_1} \cdot \frac{\delta f}{f-1}, \quad (3)$$

where  $\delta \epsilon = |\delta \epsilon_1| = |\delta \epsilon_2/f|$ , and  $\delta \epsilon$  and  $\delta f$  are the errors on fraction and ratio, respectively. For determining the number  $m$  uniquely, the condition  $|\delta m| < 1/2$  must hold.

The author is with National Research Laboratory of Metrology, 1-1-4 Umezono, Sakura-mura, Niihari-gun, Ibaraki 305, Japan.

Received 7 July 1980.

0003-6935/81/020231-04\$00.50/0.

© 1981 Optical Society of America.

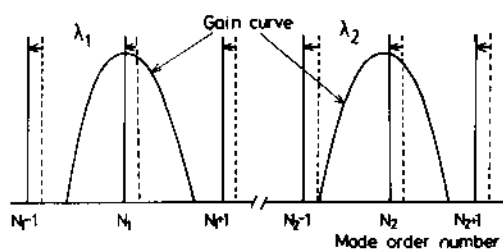


Fig. 1. Relation of laser gain curves and cavity modes.

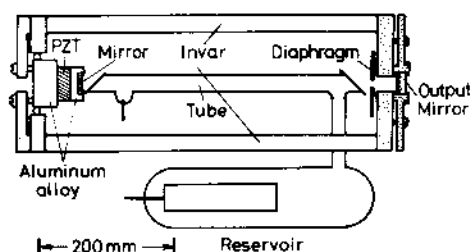


Fig. 2. Schematic of the He-Xe laser developed.

Generally speaking, as the wavelength ratio  $f$  approaches unity, the required accuracies of reading the interference order and the ratio between the laser wavelengths become more severe. On the other hand, the length determined by the excess fractions method takes longer, and the preliminary measurement by other methods becomes simpler since it equals the synthetic wavelength  $\lambda_1/(f-1)$ . Therefore, it is possible to measure long lengths.

### B. Examples

Now, applying the visible wavelengths  $\lambda_1 = 0.633 \mu\text{m}$  and  $\lambda_2 = 0.612 \mu\text{m}$  to Eq. (3) at a length of 3 m, we find the relations  $\delta\epsilon < 0.008 = 0.003 \mu\text{m}$  and  $\delta f < 2 \times 10^{-9}$  from the condition  $|\delta m| < 1/2$ . It is therefore difficult to satisfy the relations with a visible interferometer.

If  $\lambda_1 = 3.51 \mu\text{m}$  and  $\lambda_2 = 3.37 \mu\text{m}$ , we obtain the relations  $\delta\epsilon < 0.010 = 0.018 \mu\text{m}$  and  $\delta f < 1.2 \times 10^{-8}$  at 3 m. The former relation is sufficiently satisfied by the use of the infrared interferometer already reported.<sup>12</sup> The relation of the latter is somewhat severe because the two wavelengths vary independently with time. However, this is relieved if the two wavelengths simultaneously oscillate in the same cavity as shown in Fig. 1. In this case, the mode order numbers of the laser cavity  $N_i$  are approximated by

$$N_i = 2D_i/\lambda_i, \quad (4)$$

where  $D_i$  is the optical cavity length at wavelength  $\lambda_i$ . The difference  $(D_2 - D_1)$  is due to the dependence of the refractive indices on the wavelength of substances in the cavity, mainly, the fused quartz of the plasma tube windows. From Eq. (4), the wavelength ratio  $f$  is given by  $D_2N_1/D_1N_2$ . The laser cavity is designed so that change of cavity length is very small. Let the lengths  $D_1$  and  $D_2$  be linear functions of time  $t$  and change by an amount  $\Delta D$ . If the mode order numbers  $N_1$  and  $N_2$  are not changed, i.e., the change of the cavity

length is less than  $\lambda_1/4$ , the variation  $\delta f$  in  $f$  is approximated by  $f\Delta D(D_2 - D_1)/D_1D_2$ . Accordingly, the uncertainty  $\delta f/f$  is  $< 1 \times 10^{-10}$  since the difference  $(D_2 - D_1)$  is less than several micrometers, and the variation  $\Delta D$  is stabilized to less than  $\lambda_1/50$  where the cavity length is  $\sim 56$  cm. Next, if each of the mode order numbers is increased by 1 and the wavelength stabilization is carried out with  $\lambda_1$ , we obtain

$$f + \delta f = \frac{(D_1 + \Delta D + \lambda_1/2)(N_2 + 1)}{(D_2 + \Delta D + \lambda_1/2)(N_1 + 1)} \approx f \left[ 1 + \frac{\Delta D(D_2 - D_1)}{D_1D_2} + \frac{\lambda_2 - \lambda_1}{2D_2} \right]. \quad (5)$$

Accordingly, the uncertainty of  $\delta f/f$  becomes nearly  $1.3 \times 10^{-7}$ , which is reduced by the mode pulling effect.<sup>13</sup> This value is too large to achieve the method of excess fractions. However, such a change can be confirmed from the relation between the locations of the oscillation wavelengths in the gain curves, and the cavity length is piezoelectrically tuned so that the mode order numbers are returned to the original numbers (see Fig. 5). Therefore, the uncertainty  $\delta f/f$  is  $< 1 \times 10^{-10}$ .

Finally, the uncertainty of the wavelength ratio is small enough in simultaneous oscillation. The ratio must be measured to an accuracy of better than  $1.2 \times 10^{-8}$  in the air near normal conditions. This would be performed by measuring a long gauge block of known length with the infrared interferometer described later. The ratio is not influenced by the variation of the air refractive index because the refractive index of air at  $3.51 \mu\text{m}$  does not differ from that at  $3.37 \mu\text{m}$  by more than  $2 \times 10^{-8}$ , and its variation usually is  $< \pm 1 \times 10^{-5}$ .<sup>12,14</sup>

### III. Experiment

The He-Xe laser developed is shown in Figs. 2 and 3. The laser with 6 Pa of Xe and 130 Pa of He operates in the TEM<sub>00</sub> mode at the 3.51- and 3.37- $\mu\text{m}$  lines. The output power is  $\sim 0.5$  mW with a direct discharge current of 5 mA. A large gas reservoir (1.3 liters) counteracts the gas cleanup problem. The cavity, which consists of a dielectric coated flat mirror and a spherical reflector mounted on a piezoelectric translator (PZT), is supported with thermally insensitive 59-cm long Invar bars. The aluminum alloy mirror mount is designed to cancel out the thermal expansion of the Invar, so that the thermal expansion coefficient of the cavity is slightly

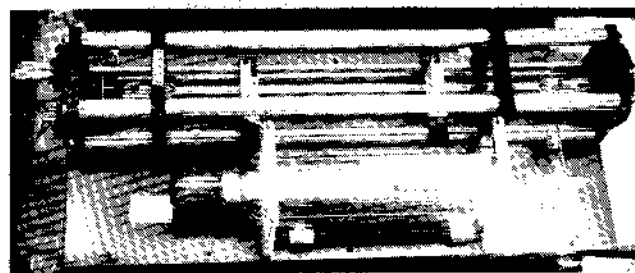


Fig. 3. View of the He-Xe laser.

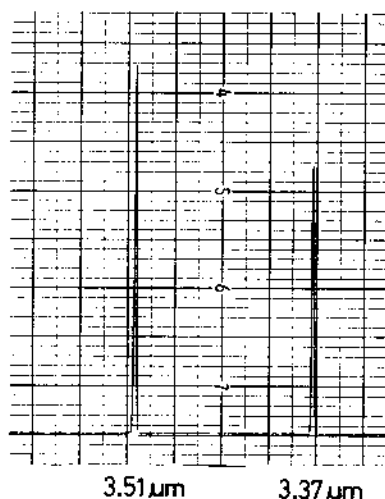


Fig. 4. Spectrum of the laser lines.

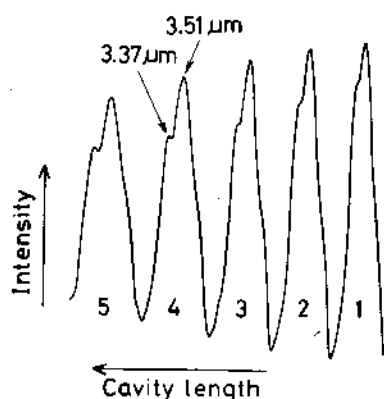


Fig. 5. Chart record of the overall power output obtained using the 3.51- and 3.37- $\mu$ m lines for tuning cavity length. The numbers indicate the sequence of the mode. Scanning is not linear.

less than that of Invar,  $\sim 1 \times 10^{-7}/^{\circ}\text{C}$ . Moreover, since the laser is room-temperature controlled, the temperature variation on the spacers is  $< \pm 0.2^{\circ}\text{C}$ . Therefore, the possibility of a change of the mode order number of the cavity is suppressed.<sup>11,14</sup> However, in an external mirror laser the variation of the refractive index of air influences the optical cavity length and cannot be neglected. Atmospheric effects, such as temperature, pressure, and humidity, are easily corrected.<sup>12,15</sup> For measuring an absolute length, one of the two lines is stabilized by piezoelectrically tuning the oscillation frequency of the line to the peak of the laser output power in the gain curve. The other line is stabilized at a definite location in its gain curve. By this method, the laser wavelengths are reproduced to an accuracy of better than  $1 \times 10^{-7}$ .<sup>9-12</sup>

Figure 4 shows a chart-recorder graph of the spectrum of the oscillation lines. The cavity length was tuned so that only the 3.51- $\mu$ m line was under the maximum power output. Figure 5 shows an overall power output of the two-wavelength He-Xe laser vs tuning cavity length. It becomes evident that the difference between the peaks of the two lines lowers each time the mode

order number is decreased by 1. From this, the mode order numbers can be confirmed if the number change is small.

Figure 6 shows an infrared two-beam interferometer for automatically measuring the length of a gauge block. A reference mirror is piezoelectrically tuned for scanning interference fringes. Gauge interference fringes and base interference fringes are detected separately by InAs detectors through a diaphragm with three 8-mm diam holes and are simultaneously recorded on a strip chart through simple operational amplifiers. A diffraction grating separates the interference fringes for the 3.51- and 3.37- $\mu$ m lines. The phase difference between the gauge and base fringes is measured with a

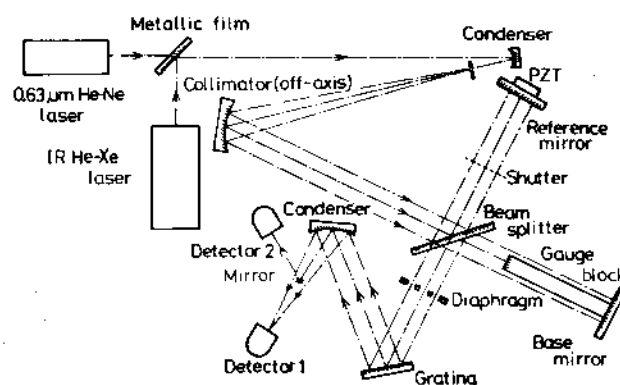


Fig. 6. Infrared interferometer for length measurement.

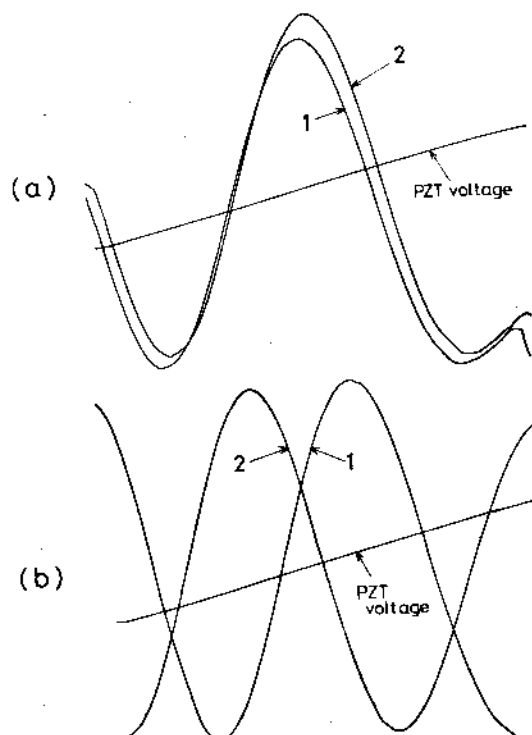


Fig. 7. Chart records of the interference fringes obtained (a) for the 3.51- $\mu$ m line, and (b) for the 3.37- $\mu$ m line. The numbers 1 and 2 show the gauge fringe and the base fringe, respectively. Pen displacement in a three-channel strip-chart recorder is not compensated.

Table I. Experimental Results

No.	Fractions		Ratio $f$
	$\epsilon_1$	$\epsilon_2$	
1	0.990	0.396	1.0416850
2	93	401	54
3	87	385	30
4	90	387	27
5	92	406	70
6	90	402	65
7	94	399	47
8	89	391	40
9	84	385	38
Mean	0.990	0.395	1.0416847
Standard deviation	0.002 <sub>9</sub>	0.007 <sub>5</sub>	0.0000013 <sub>9</sub>

resolution of better than  $1^\circ$  in phase. The nonlinearity in scan is eliminated by the use of the fringe-reading method.<sup>12,16</sup>

Using this laser interferometer, the preliminary experiment was made to confirm this method. A 7-mm long gauge block, whose absolute length was known from a visible gauge block interferometer,<sup>1</sup> was used as the standard length. Figure 7 shows chart-recorder graphs of the interference fringes obtained. The measured fractions  $\epsilon_1$  and  $\epsilon_2$  for the 3.5079 $\mu$ m and 3.3676 $\mu$ m wavelengths and the calculated wavelength ratio  $f$  are shown in Table I. It becomes evident that the errors on fractions  $\epsilon_1$  and  $\epsilon_2$  are  $<0.01$  in standard deviation. Furthermore, the error on the ratio  $f$  is half of the sum of the errors on  $\epsilon_1$  and  $\epsilon_2$  because the errors

related to both  $\epsilon_1$  and  $\epsilon_2$  are canceled. This increases the reliability in determining the length by the method of excess fractions.

The synthetic wavelength obtained using this He-Xe laser is long,  $\sim 80 \mu\text{m}$ . The measurement of lengths of several meters should be interferometrically achieved by this method with a precision of  $0.01 \mu\text{m}$ .

The author wishes to thank S. Seino for helpful discussions and suggestions.

## References

1. Y. Sakurai and S. Seino, *Bull. Natl. Res. Lab. Metrol. Tokyo* **14**, 34 (1967).
2. H. Welling and B. Wellegehausen, *Appl. Opt.* **11**, 1986 (1972).
3. P. Cerez and S. J. Bennett, *Opt. Commun.* **25**, 1079 (1979).
4. F. Spieweck, *IEEE Trans. Instrum. Meas.* **IM-27**, 398 (1978).
5. W. L. Faust, R. A. McFarlane, C. K. N. Patel, and C. G. B. Garrett, *Appl. Phys. Lett.* **1**, 85 (1962).
6. M. Ohtsu and T. Tako, *Jpn. J. Appl. Phys.* **17**, 2169 (1978).
7. C. R. Tilford, *Appl. Opt.* **16**, 1857 (1977).
8. G. L. Bourdet and A. G. Orszag, *Appl. Opt.* **18**, 225 (1979).
9. W. R. C. Rowley and D. C. Wilson, *Nature London* **200**, 745 (1963).
10. W. R. Bennett, Jr., S. F. Jacobs, J. T. LaTourrette, and P. Rabinowitz, *Appl. Phys. Lett.* **5**, 56 (1964).
11. K. D. Mielenz, R. B. Stephens, K. E. Gilliland, and K. F. Nefflen, *J. Opt. Soc. Am.* **56**, 156 (1966).
12. H. Matsumoto, *Jpn. J. Appl. Phys.* **19**, 713 (1980).
13. L. Casperson and A. Yariv, *Appl. Phys. Lett.* **17**, 259 (1970).
14. H. Nagai and I. Taniguchi, *Jpn. J. Appl. Phys.* **12**, 434 (1973).
15. B. Edlén, *Metrologia* **2**, 71 (1966).
16. H. Matsumoto, *Jpn. J. Appl. Phys.* **18**, 1661 (1979).